MATH 590: QUIZ 4 SOLUTIONS

Name:

1. True or False (no explanation required). The function $T : \mathbb{R}^3 \to \mathbb{R}$ given by $T(a, b, c) = e^{a+b+c}$ is a linear transformation. (2 points)

Solution. False.

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be given by $T(x, y, z) = (x - y, y - z, x + y - z, z - x)$. Find the matrix of T with respect to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}^4$. (4 points)

Solution. $T(1, 0, 0) = (1, 0, 1, -1), T(0, 1, 0) = (-1, 1, 1, 0), T(0, 0, 1) = (0, -1, -1, 1)$, and therefore the resulting matrix is $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$.

3. Suppose $V := P_2(\mathbb{R})$ is the vector space of real polynomials of degree less than or equal to two. Take vectors $v = 1 + 2x - 3x^2$ and $u = 2 - x + 5x^2$ in V and let $\alpha = \{1, x, x^2\}$ denote the standard basis of V. Verify that $[2v + 3u]_\alpha = 2[v]_\alpha + 3[u]_\alpha$. (4 points)

Solution. We have $[v]_\alpha = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and $[u]_\alpha = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$, so that $2[v]_\alpha + 3[u]_\alpha = 2 \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ 9 \end{pmatrix}$.
On the other hand,

$$[2v + 3u]_\alpha = [2(1 + 2x - 3x^2) + 3(2 - x + 5x^2)]_\alpha = [8 + x + 9x^2]_\alpha = \begin{pmatrix} 8 \\ 1 \\ 9 \end{pmatrix},$$

which gives what we want.